Shear Viscosity of the “semi”-QGP

1. Some possible deconfining phase transitions

2. Deconfinement at *zero* coupling: SU(∞) on a small sphere
   (Sundborg ‘99, Aharony et al ’03, ‘05)

3. Is the QCD coupling big at T_c? Maybe *not*.

4. (Renormalized) Polyakov Loops & the semi-QGP

5. Shear viscosity of the semi-QGP

For heavy ions, is LHC like RHIC?

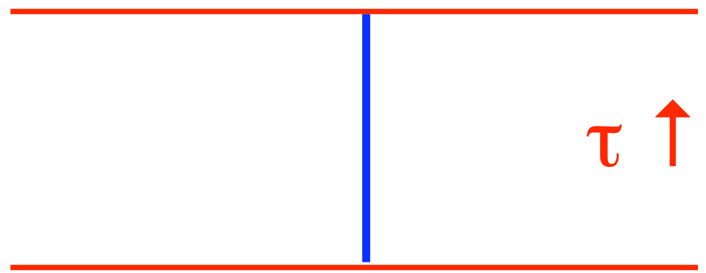
Strong-QGP, $\mathcal{N} = 4$ SUSY: *yes*.

Semi-QGP, *no*.

1. Some possible deconfining transitions

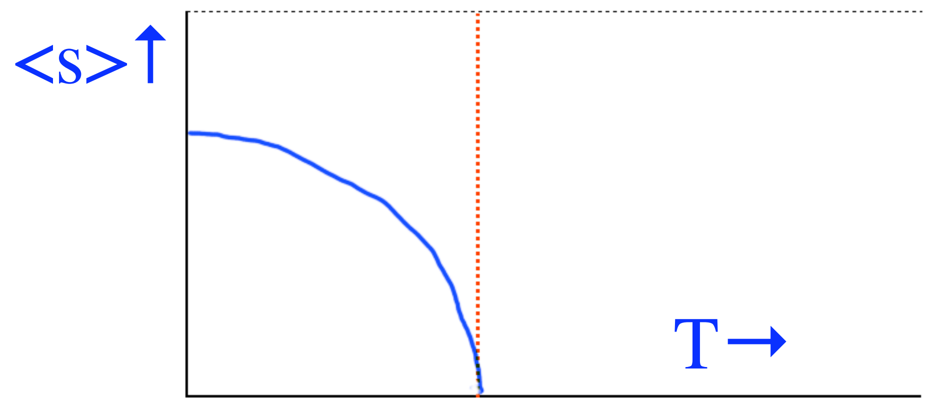
Polyakov loops & deconfinement

Polyakov loop: order parameter for deconfinement,
~ propagator of *infinitely* heavy quark



$$\ell = \frac{1}{N}\,\mathrm{tr}\,\mathcal{P}\;\exp\left(ig\int_0^{1/T} A_0\,d\tau\right)$$

Ordinary magnetization:
<s> ≠ 0 at low T, <s> = 0 at high T =>



Deconfinement: Polyakov loop "flipped",
Global Z(N) symmetry:
broken at high T, *restored* at low T.

Classify possible deconfining transitions by change in <*loop*>.

Assume overall normalization of loop physical: $\langle \ell \rangle \to 1 \;\;,\;\; T \to \infty$

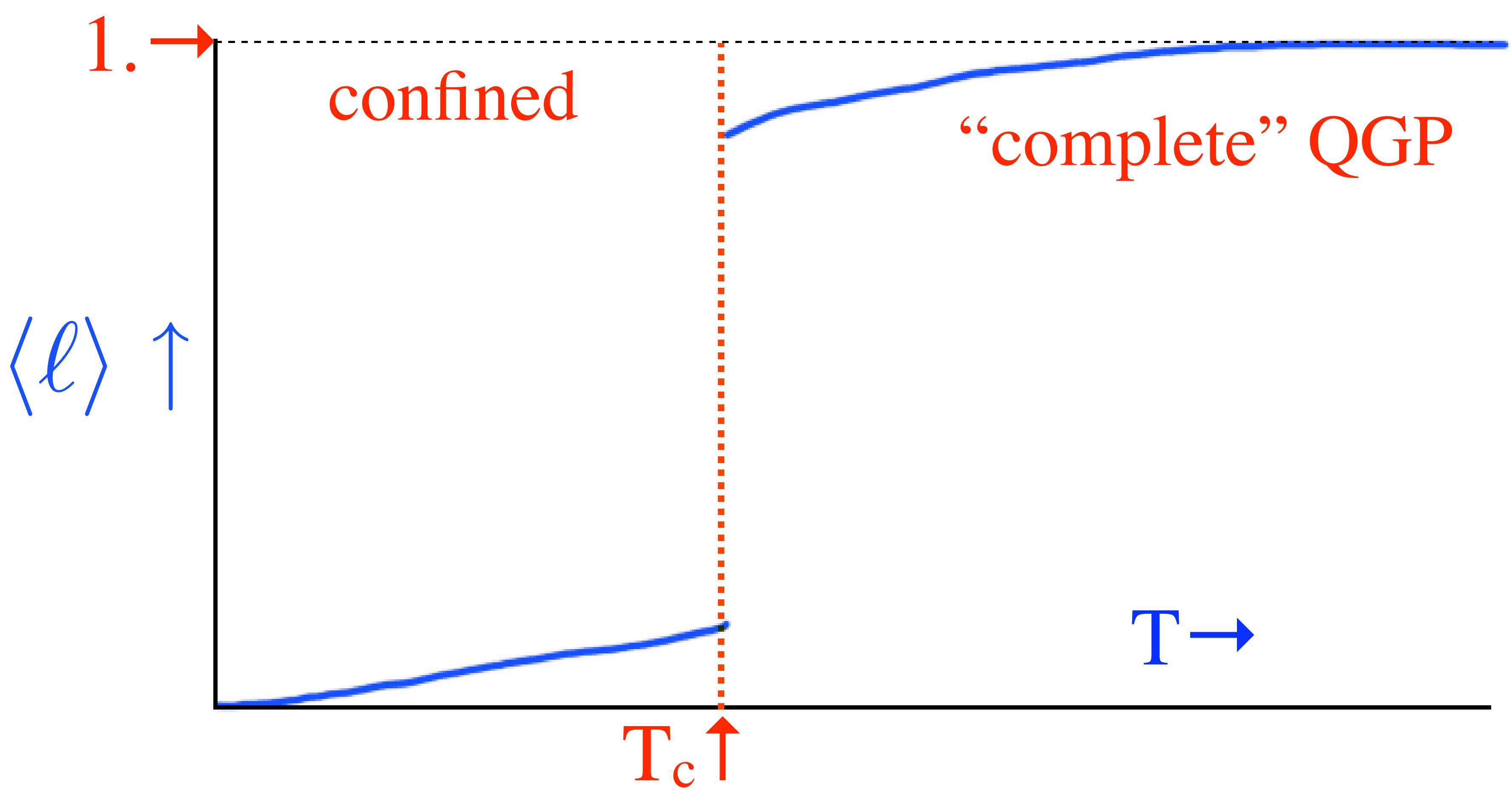
Quarks act like background Z(N) field.

One possibility

Transition from confined phase to “complete” Quark-Gluon Plasma (QGP)

Complete QGP: loop near 1, ≈ perturbative.

Transition strongly first order. Effect of quarks weak.

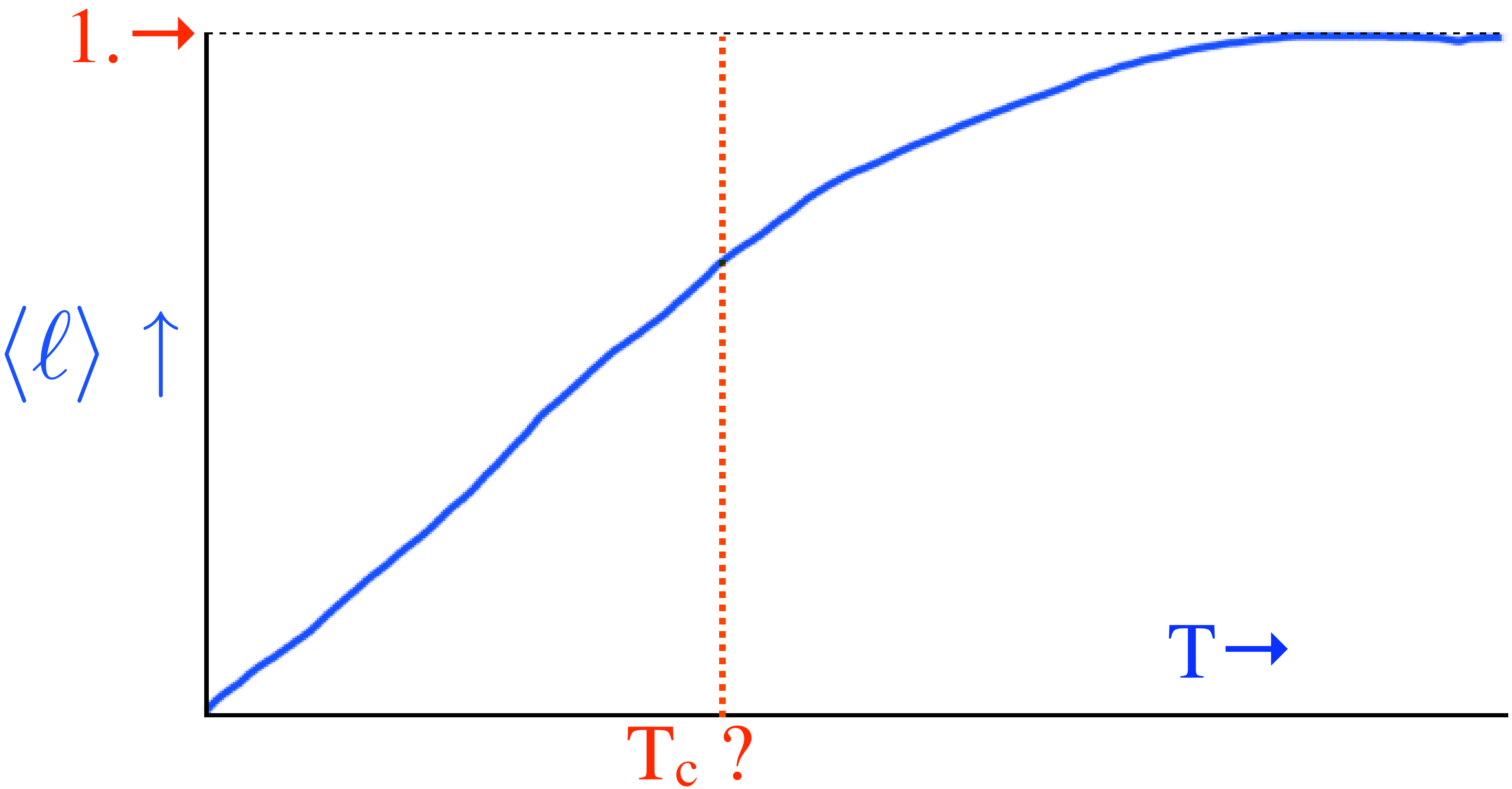


Logically possible, does *not* appear to arise in *any* context. (Lattice, analytical...)
General expectation before RHIC.

Another possibility

Many quarks, *strong* background field.

Loop increases gradually, probably no deconfining phase transition.



Probably true for large number of flavors, completely wash out deconfinement.

Probably no chiral transition either.

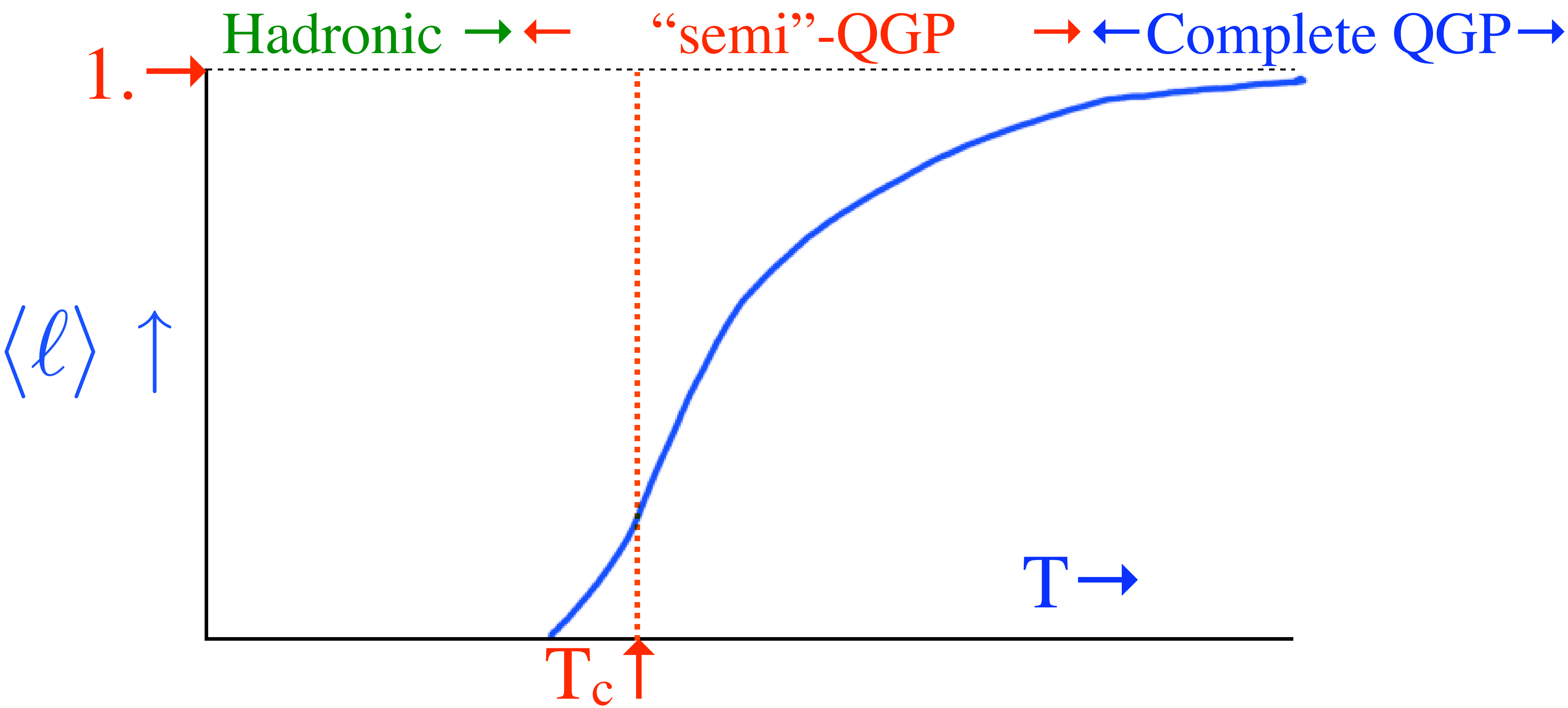
QCD?

Even with dynamical quarks, *three* regimes:

Hadronic, <*loop*> ~ 0.

"Semi"-QGP: <*loop*> nonzero, but *not* near one.

Complete QGP: <*loop*> near one. Usual "perturbative" regime (resummed!)

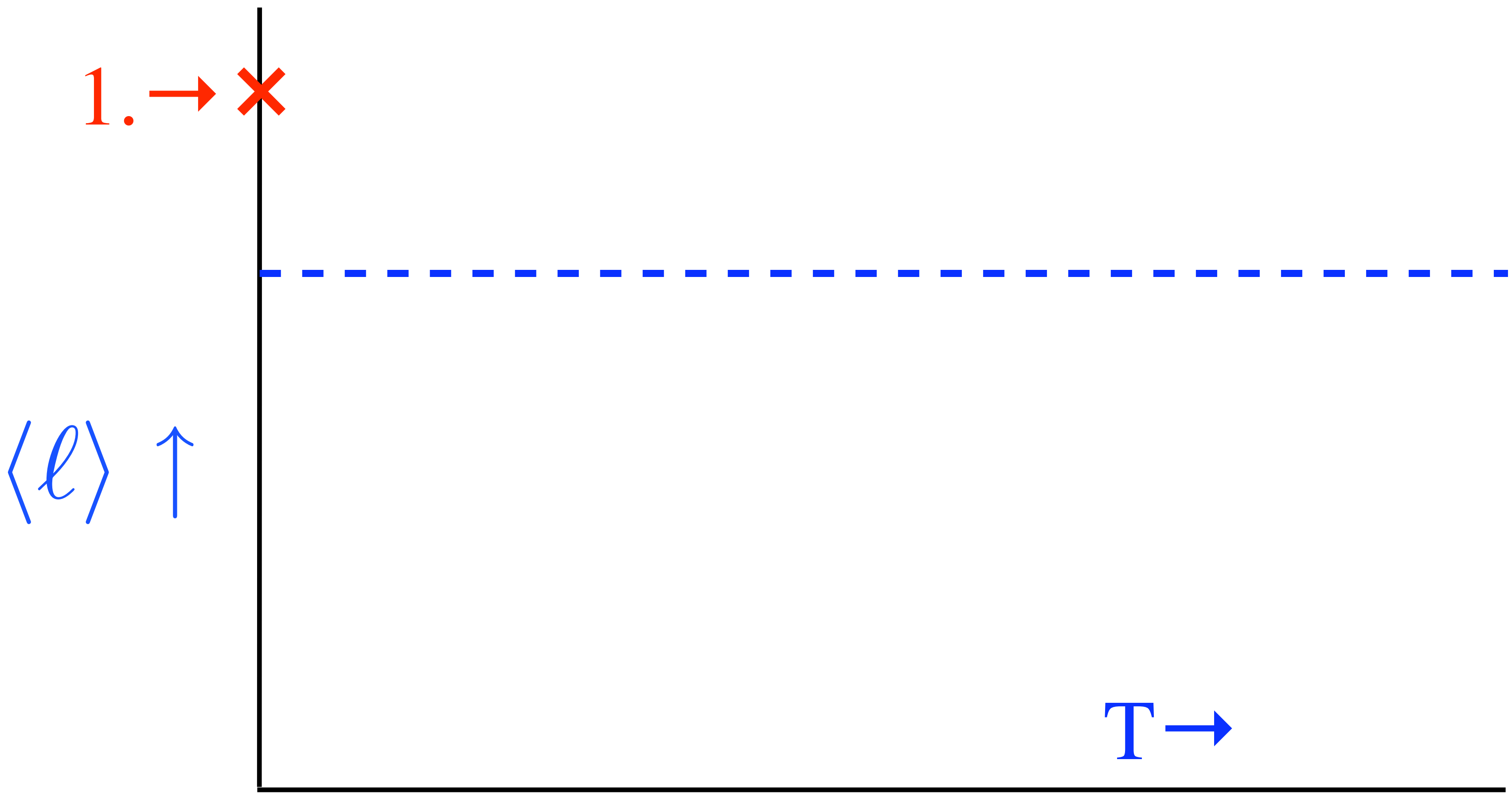


$\mathcal{N} = 4$ SU(∞)

AdS/CFT: Can define <loop> = 1 at T = 0 (Polyakov-Maldacena, + scalars)

At $T \neq 0$, *<loop>* = constant (like pressure/T^4) : value, vs g^2 N?

$\mathcal{N} = 4$ SU(∞) is *always* deconfined.



2. Deconfinement at *zero* coupling: SU(∞) on a small sphere

SU(∞) on a small sphere: Hagedorn temperature

Sundborg, hep-th/9908001
AMMPV: Aharony, Marsano, Minwalla, Papadodimas, & Van Raamsdonk,
hep-th/0310285 & 0502149

Consider SU(N) on a *very* small sphere: radius R, with $g^2(R) << 1$.
(Sphere because constant modes simple, spherically symmetric)

At $N = \infty$, can have a phase transition even in a *finite* volume.

When $g^2 = 0$: by counting gauge *singlets*, find a Hagedorn temperature, T_H :

$$\rho(E) \sim \exp(E/T_H) \quad , \quad E \to \infty$$

At $N = \infty$, Hagedorn temperature is *precisely* defined. When $g^2 = 0$,

$$T_H = \frac{1}{\log(2+\sqrt{3})} \, \frac{1}{R} \quad , \quad g^2 = 0.$$

SU(∞) on a small sphere: effective theory

Construct effective theory for low energy (constant) modes,
by integrating out high energy modes, with momenta ~ 1/R:

Consider (thermal) Wilson line:

$$\mathbf{L} = \mathcal{P}\ \exp\left(ig \int_0^{1/T} A_0\ d\tau \right)$$

L is gauge dependent,

$$\mathbf{L} \to \Omega(1/T)^\dagger\, \mathbf{L}\, \Omega(0)$$

Traces of moments gauge invariant,

$$\ell_j = \frac{1}{N}\, \mathrm{tr}\, \mathbf{L}^j \quad , \quad j = 1 \ldots (N-1)$$

Effective theory for l_j: compute free energy in *constant* background A_0 field:
Q = diagonal matrix.

$$A_0 = \frac{T}{g}\, Q \quad , \quad \mathbf{L} = \mathrm{e}^{iQ}$$

SU(∞) on a small sphere & the Polyakov loop

When $g^2 = 0$:

$$\mathcal{V}_{eff} = N^2 \left(m^2 \, \ell_1^2 + \mathcal{V}_{\rm Vdm} + \ldots \right) \quad ; \quad m^2 \sim T_H^2 - T^2$$

At the Hagedorn temperature, T_H , *only* the first mode, l_1, is unstable; all other modes are stable. Concentrate on that mode, $l \equiv l_1$.

Vandermonde determinant in measure for constant mode gives “Vdm potential”:

$$\mathcal{V}_{\rm Vdm} = + \, \ell^2 \;\; , \;\; \ell < \frac{1}{2}$$

$$\mathcal{V}_{\rm Vdm} = - \, \frac{1}{2} \, \log \left(2 \left(1 - \ell \right) \right) + \frac{1}{4} \;\; , \;\; \ell \geq \frac{1}{2}$$

Vdm potential has discontinuity of *third* order at $l = 1/2$.

Gross & Witten ’81; Kogut, Snow & Stone ’82....

Sundborg, ’99....AMMPV ’03 & ‘05

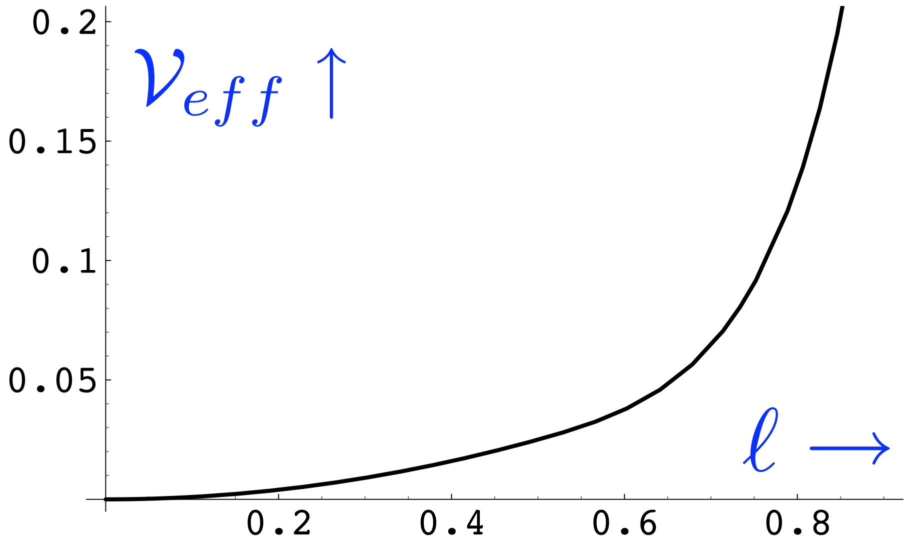
Dumitru, Hatta, Lenaghan, Orginos & RDP, hep-th/0311223 = DHLOP

Dumitru, Lenaghan & RDP, hep-ph/0410294.

Deconfinement on a small sphere

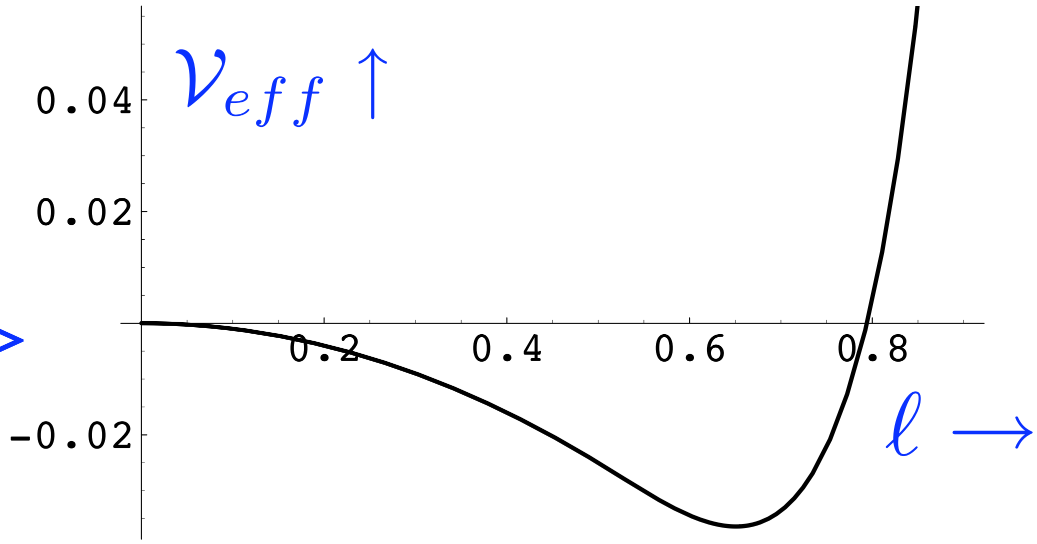
Deconfining phase transition when $m^2 = 0$: *first* order, $< l > = 1/2$ at $T_c = T_H$.

Obvious from potentials above and below T_c:



<= $m^2 = +.1$, confined phase

$m^2 = -.1$, deconfined phase =>



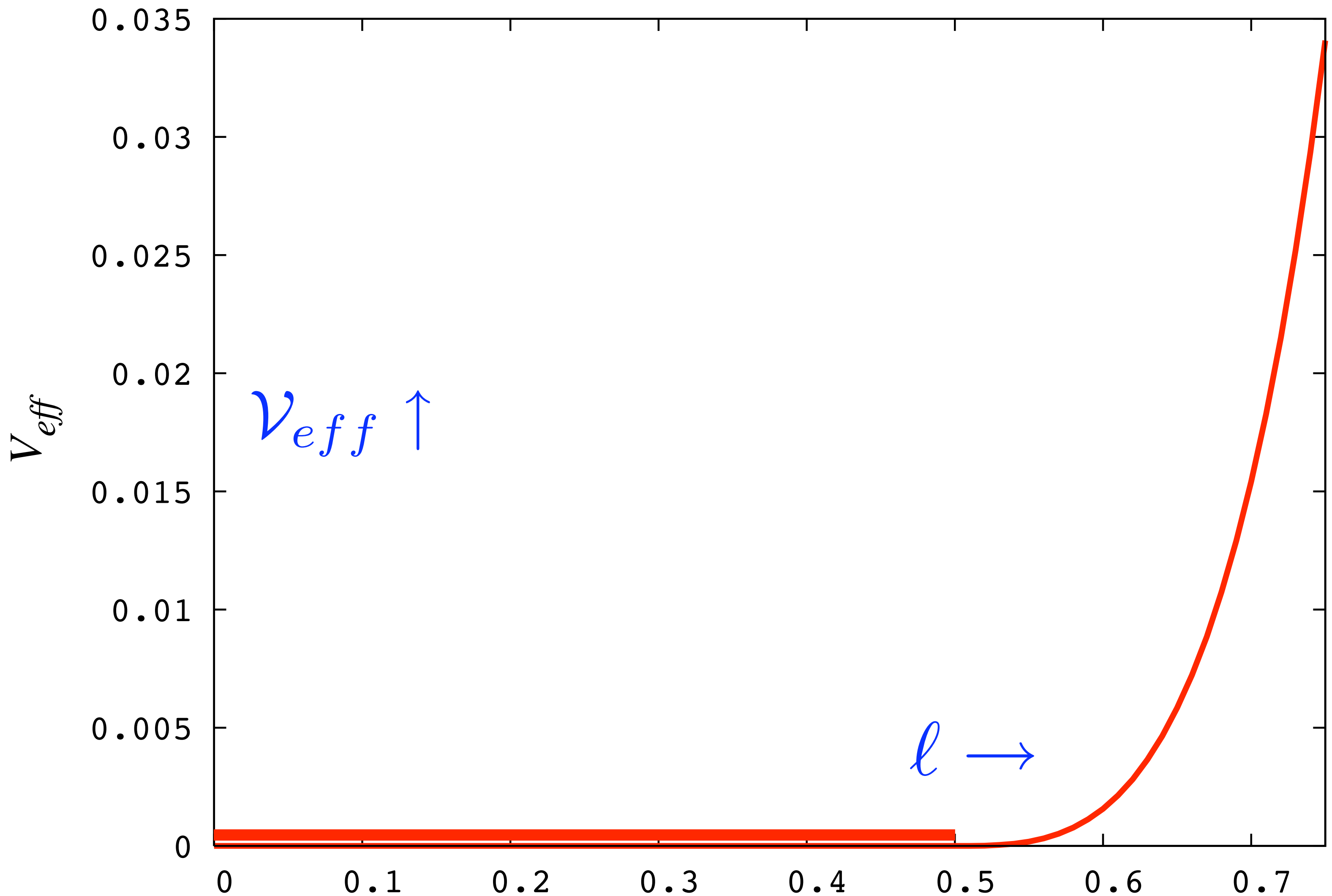
Gross-Witten point

At transition, order parameter <*loop*> jumps from 0 to 1/2. Latent heat nonzero.

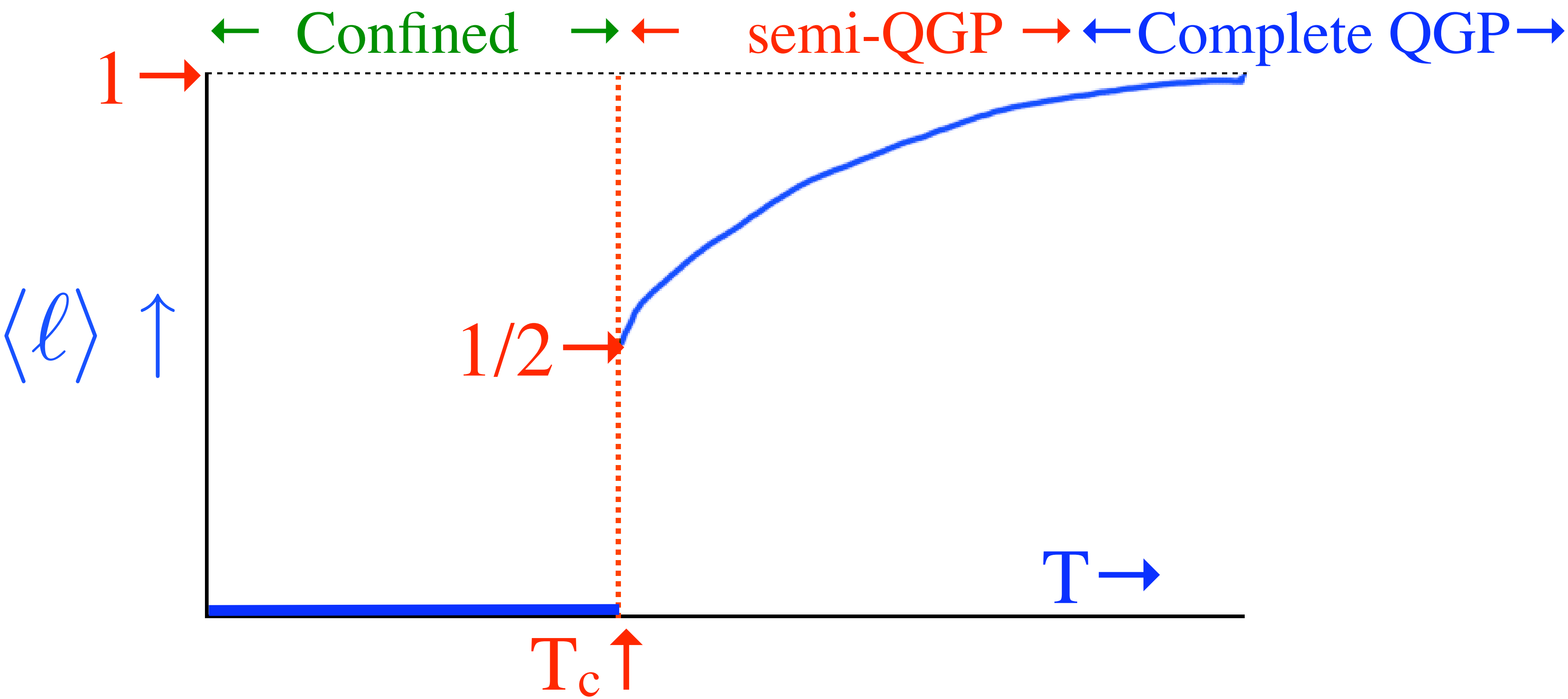
DLP: masses vanish, asymmetrically: "critical" 1st order transition: "GW point".

At $m^2 = 0$, <*loop*> jumps because of 3rd order discontinuity in Vdm potential

GW point like tricritical point in extended phase diagram.



Semi-QGP on a small sphere



Boundary between complete & semi-QGP *not* precise; <*loop*> → 1 by T ~ # T_c?

AMMPV '05: calculate free energy with Q ≠ 0 to *two* loop order at small R

$$\mathcal{V}_{eff} = \mathcal{V}_{eff}(g^2 = 0) - c_3\, g^4 \left(\ell^2\right)^2 \quad c_3 > 0.$$

$c_3 > 0 \Rightarrow T_c = T_H - O(g^4)$. Deconfinement first order, *below* T_H

3. Is α_s big at T_c? Maybe *not*.

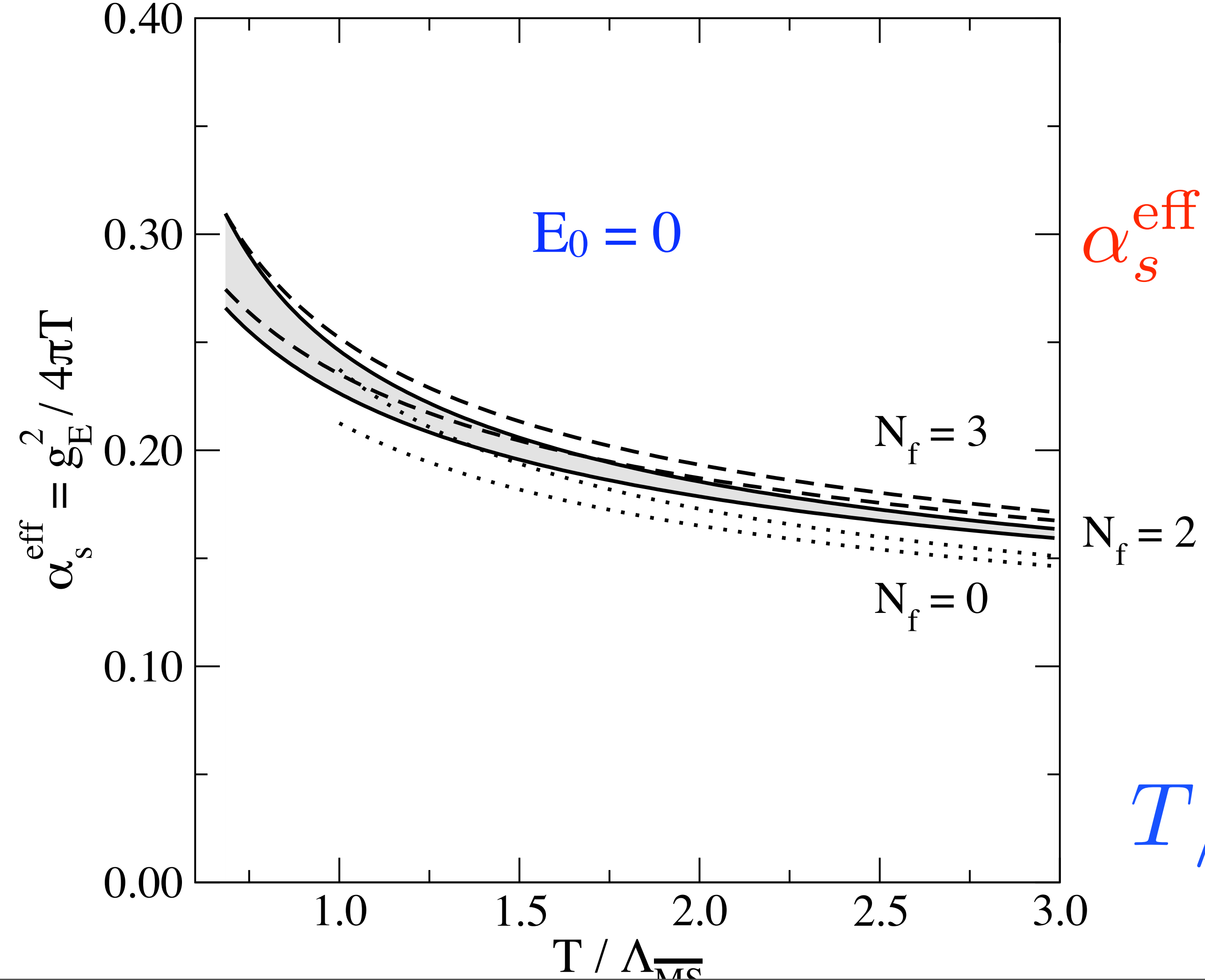
Maybe α_s is *not* so big at T_c

Laine & Schröder, hep-ph/0503061 & 0603048

$T_c \sim \Lambda_{MS} \sim 200$ MeV. But $\alpha_s^{eff}(T) \sim \alpha_s^{eff}(2\pi T) \sim 0.3$ at T_c: *not* so big

Two loop calculation: grey band uncertainty from changing scale by factor 2.

$\alpha_s^{\rm eff}(T) \uparrow$



$\alpha_s^{\rm eff}(T_c) \approx 0.3$

$T/\Lambda_{\overline{\rm MS}} \rightarrow$

Perturbative resummation of the pressure

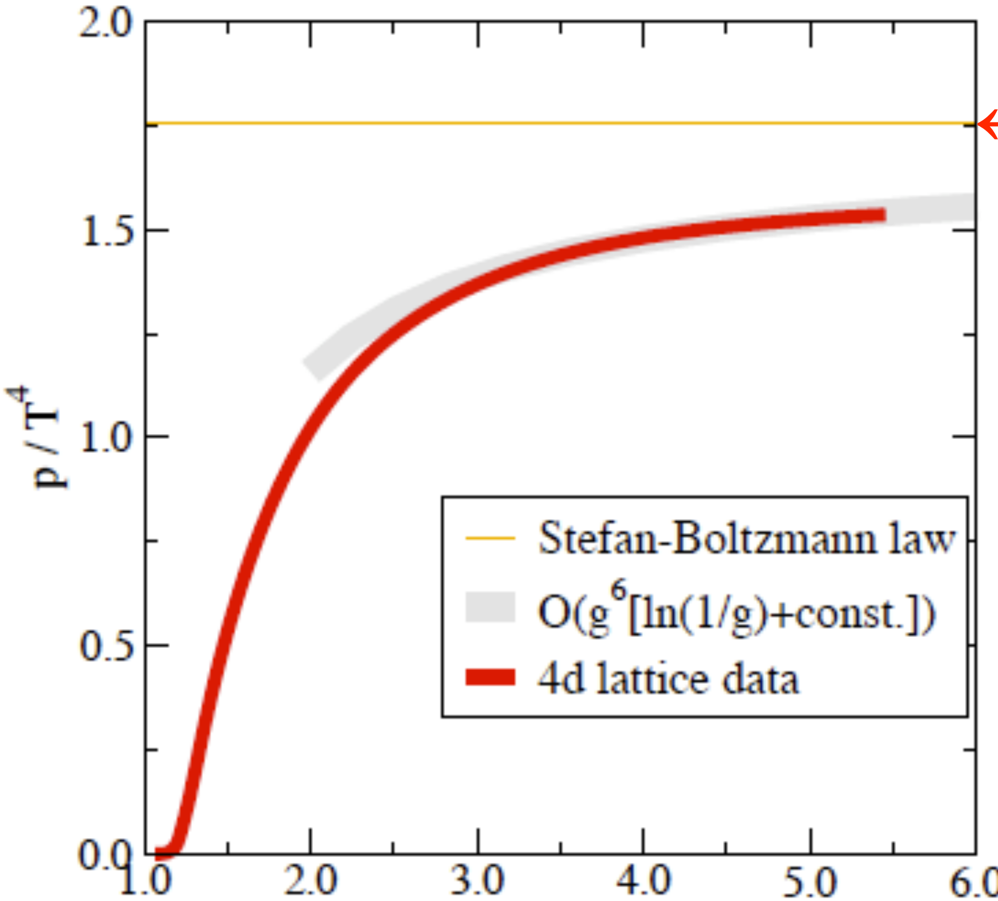
"Helsinki" resummation: Di Renzo, Laine, Schröder, Torrero, 0808.0557

$$\mathcal{L}^{eff} = \frac{1}{2}\,\mathrm{tr}\,G_{ij}^2 \;+\; \mathrm{tr}\,|D_i A_0|^2 + \; m_D^2\;\mathrm{tr}\,A_0^2 \;+\; \kappa\,\mathrm{tr}\,A_0^4$$

Now to 4 loop, ~ g^6. *Works* to ~ 3 T_c, *fails* below.

Why, if $\alpha_s^{eff}(T_c)$ is *not* so big? Perhaps a *semi*-QGP near T_c?

$p/T^4 \uparrow$



←ideal gas

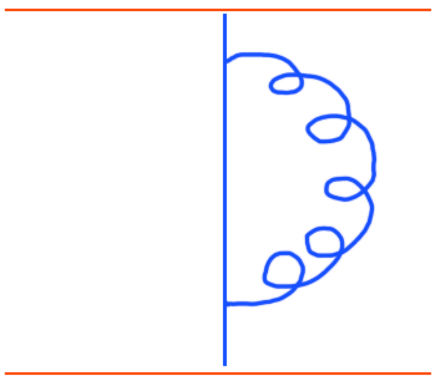
Grey: resummed pert.theory.

Red: lattice.

$T/\Lambda_{\overline{MS}} \rightarrow$

4. (Renormalized) Polyakov loops & the semi-QGP

Renormalized loops



Polyakov '80, Dotsenko & Vergeles '81...DHLOP '03...
Gupta, Hubner & Kaczmarek 0711.2251 = GHK

Bare loop UV divergent. At one loop =>
Like mass ren. of heavy quark. In 3+1 dim.'s, linear div.
Vanishes with dimensional regularization, but *not* on the lattice:

$$\langle \ell_R \rangle - 1 \sim \# \frac{C_R g^2}{T} \int^{1/a} \frac{d^3 k}{k^2} = \# \; (C_R \, g^2 + \#' g^4 + \ldots) \; \frac{1}{aT}$$

Loop in representation R, Casimir C_R.
1/(a T) = # time steps, N_t. Renormalized loop:

$$\ell_R^{\text{bare}} = \mathcal{Z}_R(g^2)^{N_t} \; \ell_R^{\text{ren}}$$

Can choose $\langle \ell \rangle \to 1 \;\; , \; T \to \infty$

GHK: find approximate Casimir scaling:
Like cusp anomalous dimension.

$$\mathcal{Z}_R(g^2) \approx \mathcal{Z}(g^2)^{C_R}$$

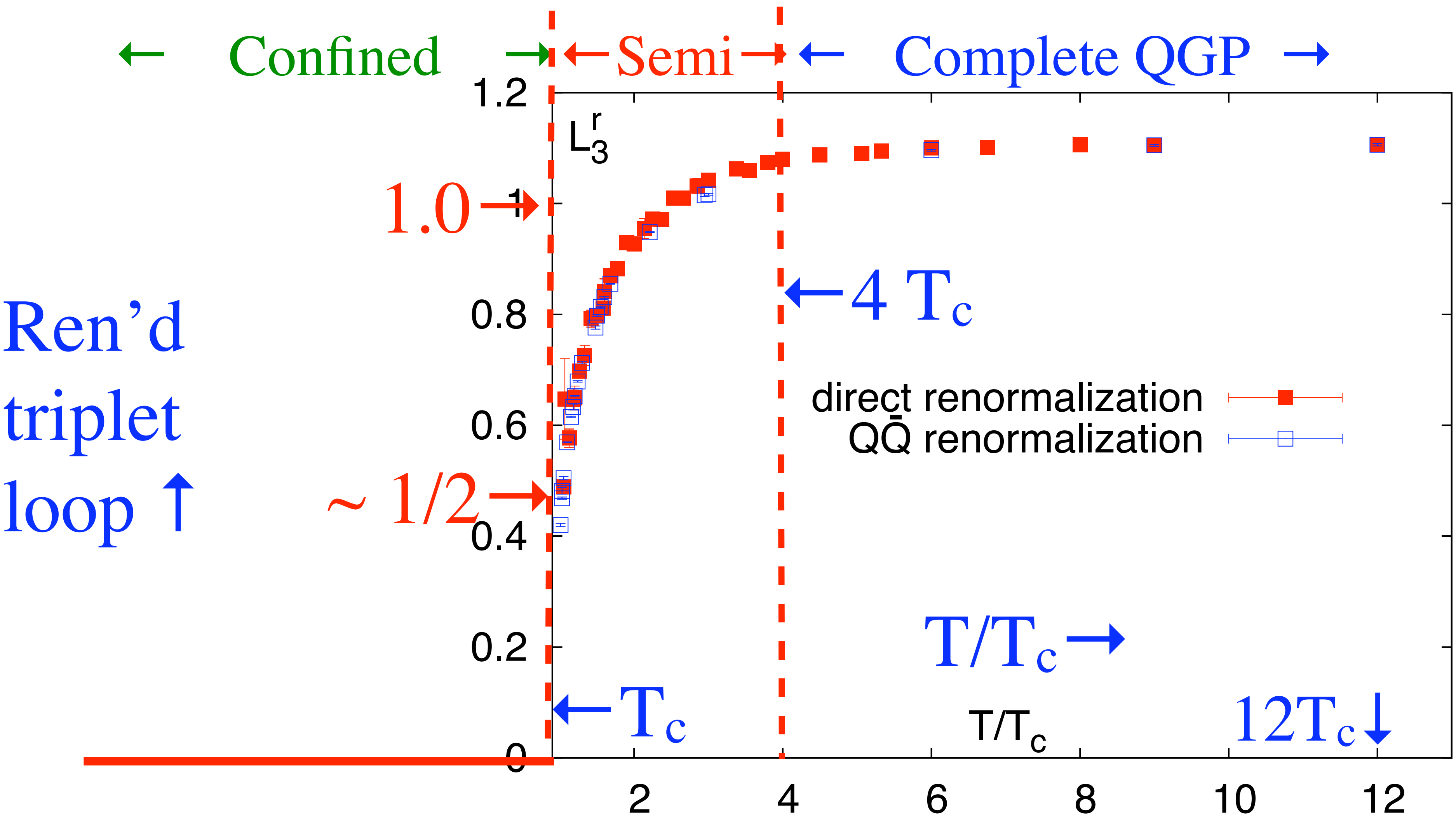
Lattice: renormalized loop, c/o quarks

GHK: Lattice SU(3), *no* quarks. Two ways of getting ren'd loop agree.

<*triplet loop*> ~ 1/2 at T_c^+! N=3 close to Gross-Witten point?

<*adjoint loop*> ~ 0.01 just below T_c . *Only* natural in matrix model.

semi-QGP: from (*exactly*) T_c^+ to 2 - 4 T_c (?). <*loop*> ~ constant above 4 T_c.

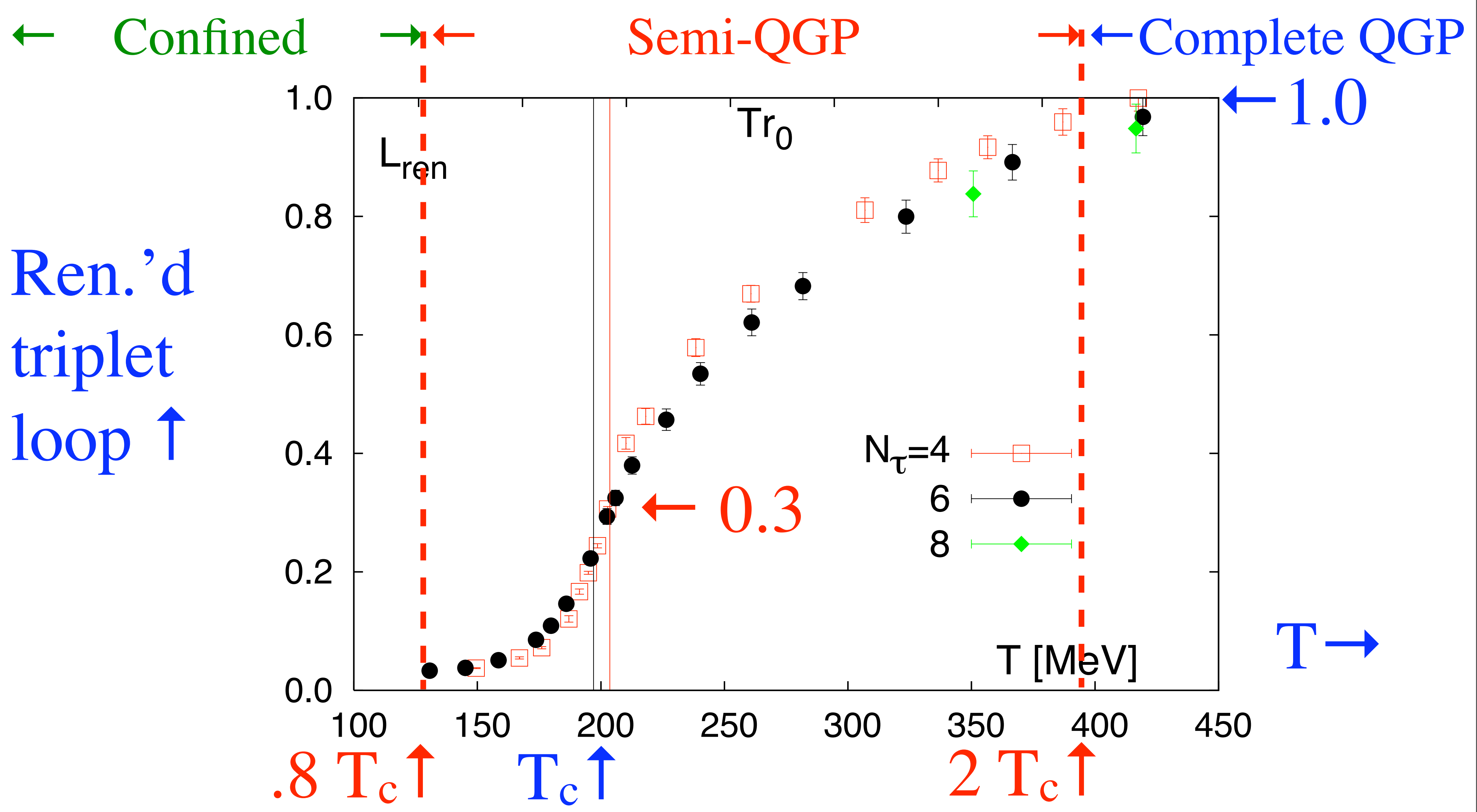


Lattice: renormalized loop, with quarks

Cheng et al, 0710.0354: ~ QCD, 2+1 flavors. T_c ~ 190 MeV, crossover.

<*loop*>: nonzero from ~ 0.8 T_c; ~ 0.3 at T_c; ~ 1.0 at 2 T_c.

Semi-QGP from ~ 0.8 T_c (*below* T_c) to ~ 2-3 T_c (?). <*loop*> *small* at T_c .



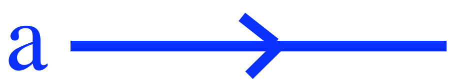
4. Shear viscosity of the semi-QGP

Semi-QGP in weak coupling

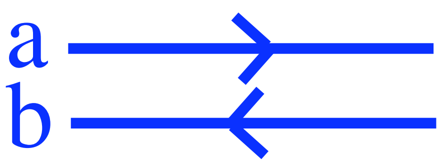
Y. Hidaka & RDP 0803.0453. Semi-classical expansion of the semi-QGP:

$$A_\mu = A_\mu^{\rm cl} + B_\mu \;\; , \;\; A_0^{\rm cl} = Q/g \;\; .$$

$Q \neq 0$: just like semi-classical calc. of ‘t Hooft loop. $Q = Q^a$, *diagonal* matrix.
Work at large N, large N_f, use double line notation. (Finite N ok, messy.)



$$i D_0^{\rm cl} = p_0 + Q^a = p_0^a$$



$$i D_0^{\rm cl} = p_0 + Q^a - Q^b = p_0^{ab}$$

Perturbation theory in B_μ’s same as $Q = 0$, but with “shifted” p_0’s.
Amplitudes in real time: $p_0{}^a \rightarrow i\,\omega$, etc. Furuuchi, hep-th/0510056

Q (imaginary) chemical potential
for (diagonal) color charge.
e.g., for quarks:

$$\widetilde{n}(E - iQ^a) = \frac{1}{e^{(E - iQ^a)/T} + 1}$$

How color evaporates in the semi-QGP

AMMPV: simple trick.

$$\mathrm{tr}\ \frac{1}{\mathrm{e}^{(E-iQ^a)/T}-1}=\mathrm{tr}\ \sum_{j=1}^{\infty}\mathrm{e}^{-j(E-iQ^a)/T}=\sum_{j=1}^{\infty}\mathrm{e}^{-jE/T}\ \mathrm{tr}\ \mathbf{L}^j$$

$L = e^{i\,Q/T}$ = Wilson line. Obtain expressions in terms of moments of L, L^j.

We *don't* know (yet) effective theory for Q's. *So we guess.*

Take first moment, l = <*loop*> = < tr L>/N, from lattice for N = 3.
For higher moments, given l, assume either: 1. Gross-Witten, or 2. step function.

L ~ propagator of *infinitely* heavy (test) quark.
In *this* semi-cl. expansion, for colored fields of *any* momentum and mass,
As $l \to 0$, *all* quarks suppressed ~ l ; *all* gluons, ~ l^2 : *universal* color evaporation

Smells right: *all* colored fields *should* evaporate as <*loop*> → 0.

Shear viscosity in the semi-QGP

Shear viscosity, η, in the complete QGP:

Arnold, Moore & Yaffe, hep-ph/0010177 & 0302165 = AMY.

Generalize to Q ≠ 0: Boltzmann equation in background field.

$$\eta = \frac{\mathcal{S}^2}{\mathcal{C}}$$

$\mathcal{S}$ = source, $\mathcal{C}$ = collision term. *Two* ways of getting small η:

"Strong" QGP, *large* coupling $\mathcal{S} \sim 1$, $\mathcal{C} \sim (\text{coupling})^2 >> 1$.

$\mathcal{N} = 4$ SU(N), $g^2 N = N = \infty$: $\eta/s = 1/4\pi$. Kovtun, Son & Starinets hep-th/0405231

"Semi" QGP: small *loop* at *moderate* coupling:

Pure glue: $\mathcal{S} \sim$ <*loop*>^{2}, $\mathcal{C} \sim g^4$ <*loop*>^{2}

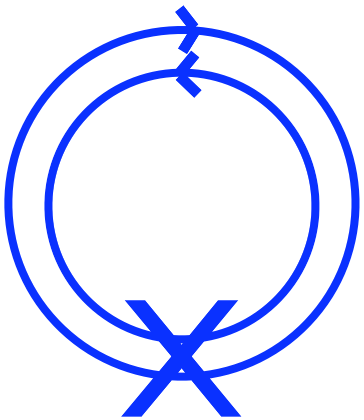
With quarks: $\mathcal{S} \sim$ <*loop*>, $\mathcal{C} \sim g^4$

Both: η ~ <*loop*>^{2}

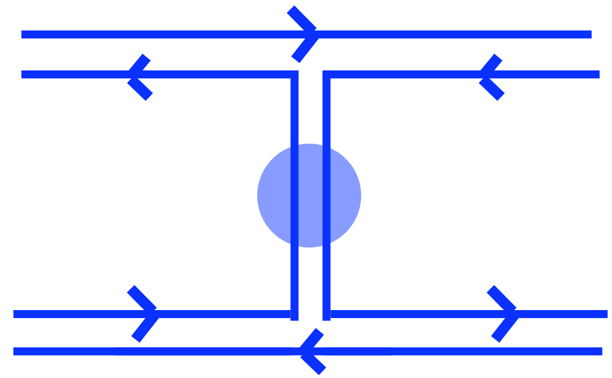
To leading log order: # from AMY, constant "c" beyond leading log

$$\frac{\eta}{T^3} = \frac{\#}{g^4 \log(c/g)} \mathcal{R}(\ell) \quad ; \quad \mathcal{R}(\ell \to 0) \sim \ell^2$$

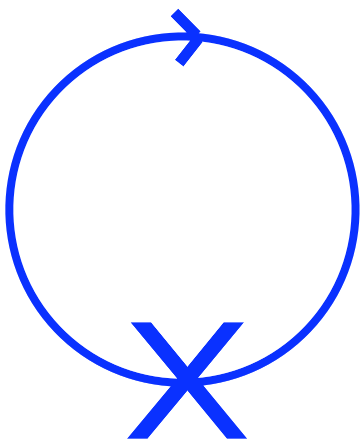
Counting powers of <*loop*> = *l* → 0



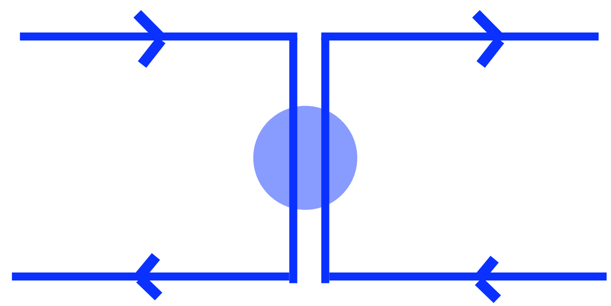
$\mathcal{S} \sim \ell^2$



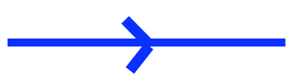
$\mathcal{C} \sim \ell^2$



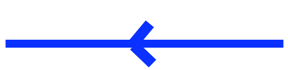
$\mathcal{S} \sim \ell$



$\mathcal{C} \sim 1$



$\sim e^{+iQ^a/T}$



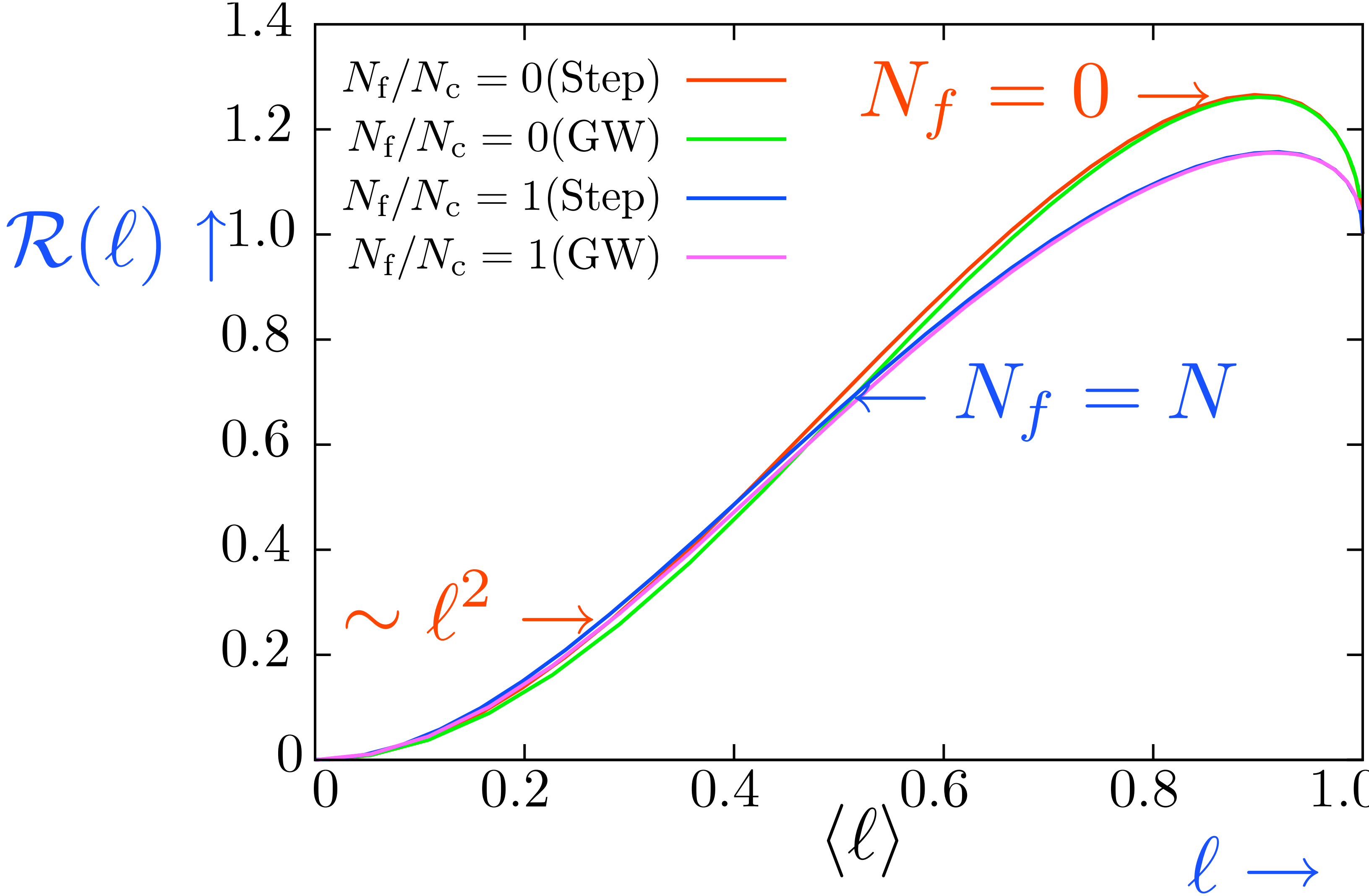
$\sim e^{-iQ^a/T}$

Small shear viscosity from color evaporation

R = ratio of shear viscosity in semi-QGP/complete-QGP at *same* g, T.

Two different eigenvalue distributions give *very* similar results!

When <*loop*> ~ 0.3, *R* ~ 0.3.



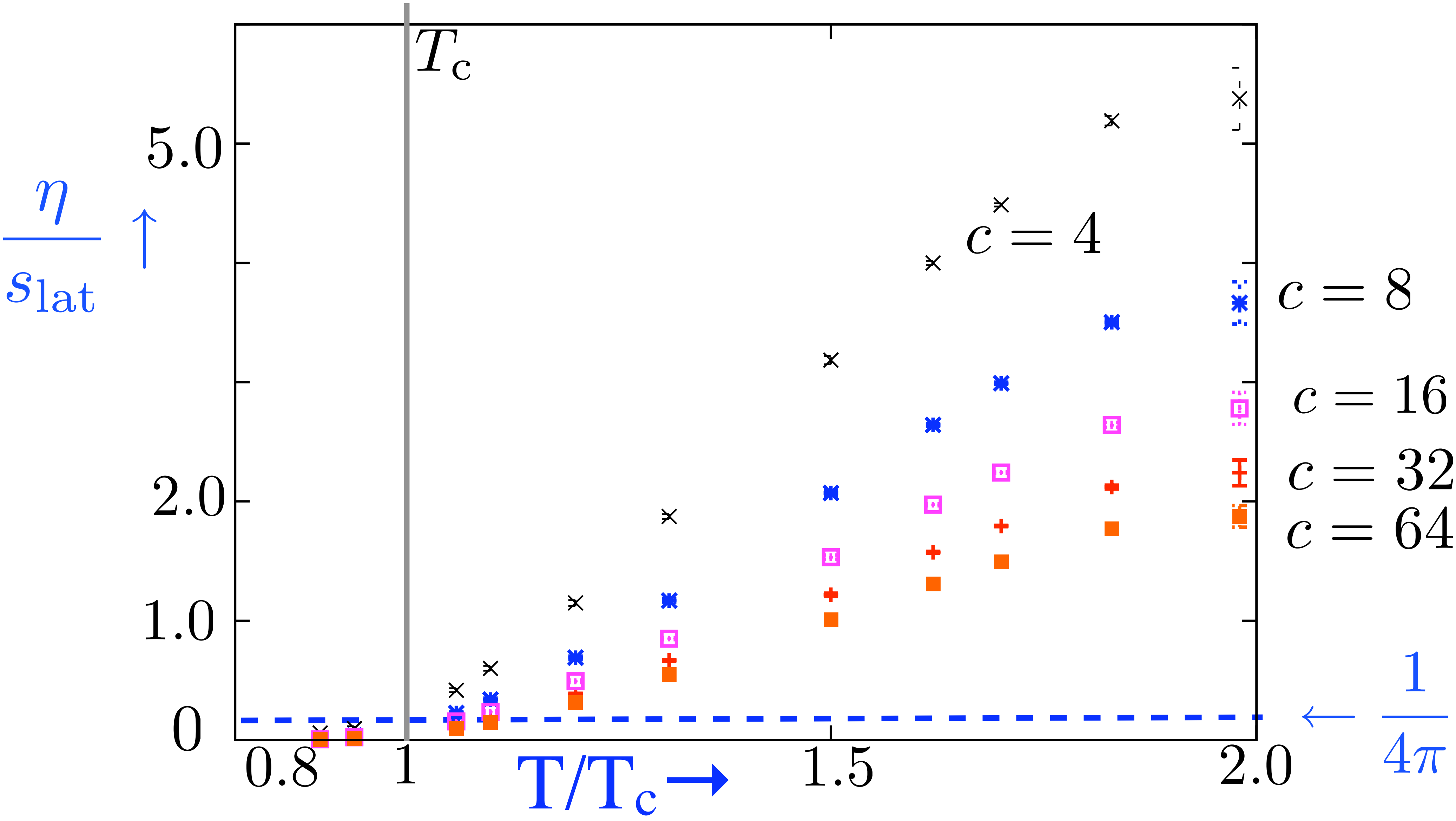
←Cusp near 1: smoothed out by Q ~ g T?

Shear viscosity/entropy

Leading log shear viscosity/lattice entropy. $\alpha_s(T_c) \sim 0.3$.

Large increase from T_c to 2 T_c. *Clearly* need results beyond leading log.

Also need to include: quarks and gluons *below* T_c, hadrons *above* T_c. *Not* easy.



Strong- vs. Semi-QGP at the LHC

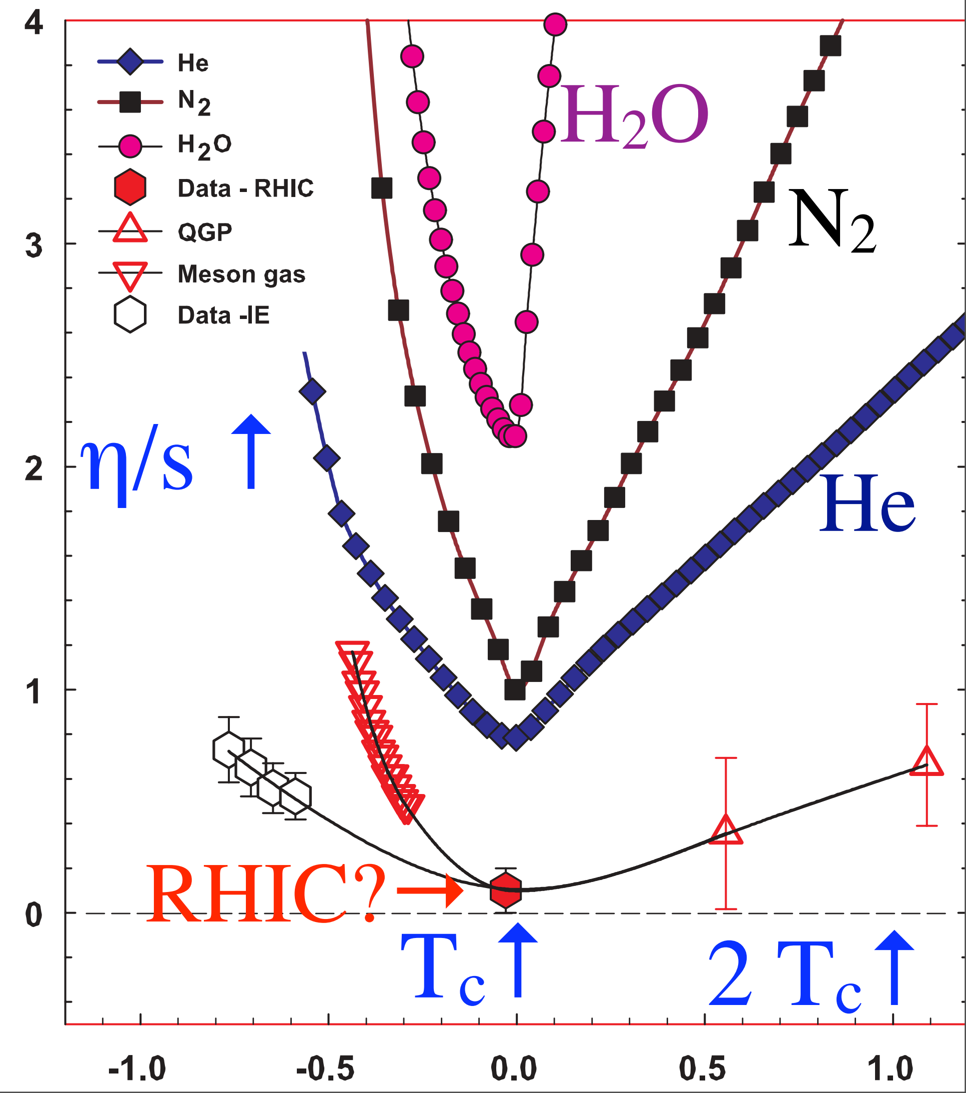
At RHIC, $\eta/s \sim 0.1 \pm 0.1$
Luzum & Romatschke, 0804.4015

Close to $\mathcal{N} = 4$ SU(∞), $\eta/s = 1/(4\pi)$.

Strong-QGP: in $\mathcal{N} = 4$ SU(∞),
add scalar potential to fit lattice pressure
But η/s *remains* $= 1/4\pi$!
Evans & Threlfall, 0805.0956
Gubser & Nellore, 0804.0434
Gursoy, Kiritsis, Mazzanti & Nitti 0804.0899
So LHC nearly ideal, like RHIC.

Semi-QGP, *and* non-relativistic systems →
Large change in η/s from T_c to 2 T_c.
At *early* times, LHC *viscous*,
*un*like RHIC

Lacey, Ajitnand, Alexander, Chung, Holzman, Issah, Taranenko, Danielewicz & Stocker, nucl-ex/0609025 ↓



Zero point energy & renormalized loops

Renormalization valid for arbitrary Wilson loops:

$$\mathcal{W} = \mathrm{tr}\, \mathcal{P}\, e^{ig \oint A_\mu dx^\mu} \quad ; \quad \mathcal{W}_{\mathrm{bare}} = \mathcal{Z}_{\mathrm{div}}\, \mathcal{W}_{\mathrm{ren}}$$

Two ambiguities:

$$\mathcal{Z}_{\mathrm{div}} = \mathrm{e}^{E_0 L}\, \mathcal{Z}_0\, \mathcal{Z}(g^2 \ldots)^{L/a} \quad ; \quad \mathcal{W}_{\mathrm{ren}} \to \mathrm{e}^{-E_0 L}\, \mathcal{Z}_0^{-1}\, \mathcal{W}_{\mathrm{ren}}$$

Overall scale trivial: $Z_0 = 1$ by requiring <*loop*> → 1 as T →∞.

E_0 = ground state energy for potential from Wilson loop: $E_0 = \# \sqrt{\sigma}$. # ?
Can *define* $E_0 = 0$ order by order in perturbation theory with *any* regulator.
$E_0 = 0$ also in string model: Nambu-Goto *plus* extrinsic curvature terms...

Lattice provides *non*-perturbative way to *define* $E_0 = 0$.
Is $E_0 = 0$ a choice, or new condition for renormalizing non-local operators?

T = 0 potential with dynamical quarks: can define *energy* for string breaking.